**The art of building a loudspeaker to the end.**

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This very ambitious title is meant in this way - you define the end, and I deliver the knowledge of, how to reach that end, by using the directions described in the following text and the text from “Components”, “How to build to the end” and “Loudspeakers in practise”

With sound reproduction it always ends up as a ***matter of taste*** it seems – why?
You are surrounded with a sea of sound waves from birth to death. The first many years the sound comes from natural sources. The main task in this period is to learn to connect the sound with its source and get meaning from these sounds***. What you learn then is the base for your hearing the rest of your life.***
Experience with music for every single person is built up from a mix of playing yourself, listening to others playing or listening to loudspeakers or when reached a certain age concerts either acoustical or amplified.
These many ways are so different in the resulting sound, that it is normal to listen to the contents of the music and not so much to the quality of the reproduction. It just has to be reasonably good and that means normally inferior.
For others and me the music only reach us if the reproduction is up to the highest standard concerning distortion in all its aspects.
Building to the final end, will be a rather ***complicated process***, as it involves rethinking of all used components and technologies without exception, but less will normally do.
Normaly used methods and components have been tried, and one by one, through the years, listened to, questioned, examined and discharged. This strange behaviour has forced me to find other ways. But first and foremost to find out why sound from loudspeakers always can be detected as coming from exactly that.
It has been a tedious and very expensive process, but also exciting and rewarding.
Fortunately I have been repairing defect loudspeaker units for many years. Thereby I have had opportunities to toy with, rebuild and change that very important part of them all. Here I must thank the loudspeaker manufacturer “Scan Speak” for their willingness to help this never satisfied old grumbler.
I have also been lucky to establish close connection with The Tobias Jensen factory www:inet.uni2.dk/~tobjen , whose expert knowledge in capacitors, in winding techniques and vacuum impregnation has lead to development of a new audio capacitors, air coils and hopefully new resistors for use in loudspeaker crossovers.
Building loudspeakers has been a hobby of mine for nearly 25 years.
All energy has been thrown into the ***search for a loudspeaker,*** which could reveal all information hidden in the program material, whatever that might be. Sound reproduced in such a way, that you, actually with your eyes closed, can see the events stored for you - not only listen to, but almost experience as real actors and instruments.
To achieve that, it has been necessary to examine every part, from the loudspeaker units and backwards to the power socket, to find the sources of shortcomings, and really!

There are many.
As a DIY-man, now mid fifty, it has been possible - with no financial director to stop me - to develop new components and technologies purely for audio purpose.
It hasn't come easy, I can tell you, and has taken far too many years. First to hear, then to find the source of the disturbing parameters, and finally to develop new ways without them.
I'm too old to start production of all these findings. It is therefore much more reasonable to tell about them, and thereby inspire others to produce or use these findings.
The internet gives opportunities to communicate world wide with these rather few people, who realise, that building a good loudspeaker is much more, than to fix a box from 6 pieces of wood, put dampening material within, and place loudspeaker units on the front, equipped with as few filter components as possible. You just have to call it hi-fi and make it to look nice and you are a manufacturer of loudspeakers.
That is really what you get even from the expensive part of the market. No real innovations, none new ways, as if normal methods are the best - but designs and finish are very impressive. Sadly ***this is for the eyes not for the ears***.
It seems difficult to build loudspeakers, which are more than 2-way, if you search the market. What has been gained, by these simple 2-ways constructions concerning recreation of a convincingly holographic picture of sound is lost, when more ways are chosen. Just consider how much energy has been put into the two way systems, to make them full range. None of them reaches the goal.
Close friends, who know to handle such things, have also done findings of faulty components and circuits in amplifiers, preamplifiers, CD-players and converters. My role in this has been to throw some new ideas into the arena and then listen to the changes and turn the thumb up or down. But also to keep the antenna high and receive the hidden messages, as ***it isn't so straight forward as it may seem***, even though the rules to follow are rather forthright, just hard to follow in practise. Habits of thinking and manufacturing are hard to fight.

**The dividing network.**
The main invention for a multiway loudspeaker to work properly at all, is a new approach to the dividing network.
This is needed if ways of building 2, 3, 4 and 5 way loudspeakers with the same manner of reproduction must be found.
In theory more ways should lead to better results, but practise has shown it otherwise. Normal filters and components are insufficient.
There is no way around the dividing network. There hasn't so far been made a fullrange loudspeaker, which can reproduce the dynamics and transparency present at a realistic full range level. It has been tried for I don't know how many years now, with none real break through.
With a well-constructed horn and a single loudspeaker, much can be achieved concerning experience, but full range it will never be. But they can be very spellbinding.
I have never really understood, why a divided system, where every single unit is optimised to its working area, suffer from absence of the ability, to take you by heart, and let you experience the event. The lack of that has driven me to ***construct and build***

***again and again***, in order to achieve that goal of excitement, fright, delight and nearness, possible to reach, when the stored music is brought to life in the right manner. In that search, the dividing network at first seemed to be the culprit, but there were much - much more to come.
At the time for development of this, the personal computer came into economical reach, and the search started by the tiny ZX81. In use of this little machine the first solution came forward.
Its characteristics of linear frequency response, zero phase turn and possibility of linear impedance, the last determined by specific loudspeaker loads, got me hooked for many years, trying to use that theory with real loudspeakers. They could something very right. I learned a lot, but I couldn't achieve, what I really wanted from that theory.
One Sunday afternoon I again toyed with the formulas - now on a QL, which I still use, when I by the merest chance found another solution hidden within the mathematics for the network, forcing all units to work in phase with each other, and much easier to realise. It had been there before my very eyes, in all that many years, and I hadn't seen it.
The work from there was to find how this solution could be derived from simpler equations and it all ended up forming a brand new family of dividing networks, which luckily contained well-known members.

**The dividing network in general**
It can be said, that ordinary calculations on dividing network have no interest in the real world. They require linear impedance, linear frequency response and furthermore equal dispersion of sound to work properly. No unit can fulfil these demands, but it doesn't really matter. Because

- by dividing the frequency area in suitable small parts,
- and furthermore use a network, that forces the differently sized units to play in phase with one another at all frequencies,
- then let the units have great overlap
- And let the high level of the units be where they work without circular break-up (the diameter of the diaphragm must be smaller than ½ the wavelength *).
- Then it all will work together to form, what is best described as ***one single unit***.

* *Normally even that ½ wavelength isn't small enough, as the first irregularity will occur at around 1/6 the wavelength, but by a special construction of the diaphragm it can be stretched to ½ the wavelength - even further. The resonance frequency must also be lowered and be kept at a Qt < 0.5.*

First we must find the calculated response curves, which can be met, for a single unit. Then use ***them*** in search for the missing electrical filter parts we must add to the mechanically filter within the unit.
We know from the mathematics, exactly what the single unit shall do - we have the target for each of them and also for the whole system..
If the unit is perfectly manufactured, it will ***turn out to be a bandpass function*** with second order slope in both ends. It will also act as a ***minimum phase network***, so

electrical and mechanical filtering is equal. In the real world you will normally have a problematic first break-up point, which need to be taken care of, but that will be discussed later in “How to build to the limit”.

**Development of a new filter theory for loudspeakers.**
In the literature you’ll find “the perfect filter” described as having linear response until a sudden abrupt cut-off. That is probably correct concerning telecommunication but it ***can’t be more wrong*** with loudspeakers. There the cut-off must happen in a very gentle manner, so let us see, how this can be achieved.
Here I can't avoid mathematics, but it is rather straightforward so jump out into it.

Special signs are used: * multiplication, / division,

It has been shown, that when you are working in the complex s-plane, in order to get linear phase and amplitude, the formulas for each unit in the dividing network must add up ***to give exactly one***.
This is so for the expression: 1. H(s) = (1 + s)/(1 + s)
It equals one for any value of s.
S is the complex frequency (jω)=j*2*pi*f and H(s) is the frequency response in volt.
The "j" is a special number = the square root of minus one. It can be treated as other numbers, and will come forward as a whole number when an even exponent powers it, else it plays its main role in turning the phase.

The expression (1.) can be split into two parts:
Lowpass: 1/ (1 + s) smaller when s gets bigger and equals 1 when s is very small
Highpass: s/ (1 + s) smaller when s gets smaller and equals 1 when s is very big

When you let ω in s=jω vary from zero to infinity, you have expressions for a well-known filter. In some circles it is appointed as the only usable kind of network, 2 way first order.
Remark that both expressions are ½ when s=1, which is the dividing frequency.
This filter has a peculiar quality, as it doesn’t matter, how the units are connected - you may choose - in phase or antiphase.
For the last mentioned the expression is: 2.H(s)= (1 - s)/(1 + s)) (allpass-version)

The two versions sound alike and differ only in their total phase response, not clearly heard.
***This filter will never work properly despite its good reputation***. It demands units from another world. It is too simple. Remember that units are bandpass functions of second order by nature, so you really need amplification, if first order filtering should be realised, else the result will be of 3rd order.

To get higher order you can square the expressions. (1*1=1)
1a. 1.*1. H(s) = ( $s^2$ + 2s +1 )/( $s^2$ +2s + 1 ) and

2a. 2.*2. $H(s) = ( s^2 - 2s +1 )/( s^2 +2s + 1 )$ ***but you can also***
3a. 1.*2. $H(s) = (1+s)(1- s) /( s^2 +2s +1 ) = (1 – s^2)/(s^2 +2s +1)$

1a. and 2a. sound again alike, they just differ in phase (+ or -).
They have 3 parts in the nominator, and thereby lead to a 3-way system - second order for highpass ($s^2$) and lowpass (1), but first order for the bandpass (2s).
It is possible to split the three parts into a 2-way version (quasi complementary), but I prefer the clear form, where every single part represents one unit.
B&O used the first expression - with a slight modification - in their famous fillerdrive system. That was the first attempt to look at all three units simultaneously. They left it again, probably because the demands for the bandpass unit are very high regarding bandwidth and sound pressure.
The third - in some papers called second order Linqwitz -Riley, is also well known for its linear frequency response and its mild turn of phase. This is equal for both low- and high pass in slope, but with a constant difference at 180 degrees. This is easy to fix by turning the phase of one of the units.
***This filter is as such very good***, the best when you want a 2-way loudspeaker.
The units can't fulfil the demands of the filter, as it is only 12 dB. You therefore must nullify the electrical filter function, when the units' mechanical filter takes over. This can be done by resistors, parallel with serial components and serial with parallel components, and will work most beautifully.
What may interest you here, are the curves for the amplitude of the two units.
To make is easy for you to plot the slopes; the filter damping is given in two scales for you to mark on an empty measuring paper.
***Second order two ways - Allpass (named Linqwitz-Riley)***

Dividing frequency: 1 kHz, but can after drawing be copied up or down in frequency.

**1/3 oktav Paper**

| Frq | Low dB | High dB |
|---|---|---|
| 19.7 | 0 | -68.24 |
| 24.8 | 0 | -64.23 |
| 31.3 | 0 | -60.21 |
| 39.4 | -0.01 | -56.21 |
| 49.6 | -0.02 | -52.2 |
| 62.5 | -0.03 | -48.2 |
| 78.7 | -0.05 | -44.2 |
| 99.2 | -0.085 | -40.2 |
| 125 | -0.135 | -36.26 |
| 157 | -0.21 | -32.32 |
| 198 | -0.335 | -28.43 |
| 250 | -0.53 | -24.61 |
| 315 | -0.82 | -20.89 |
| 397 | -1.27 | -17.33 |

**Normal paper**

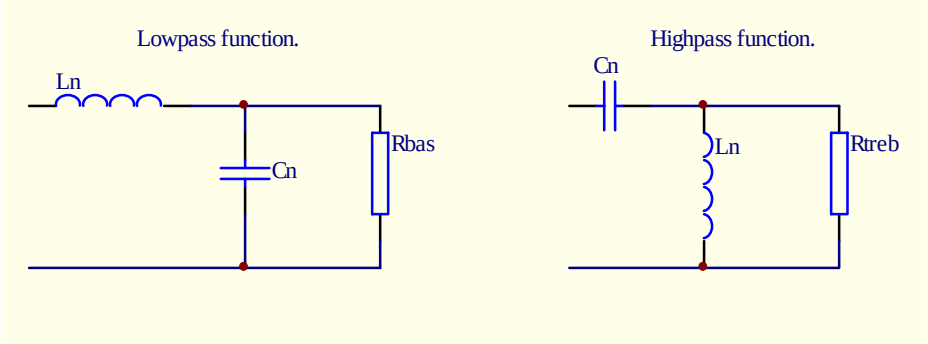
| Frq | Low dB | high dB |
|---|---|---|
| 20 | 0 | -67.96 |
| 30 | 0 | -60.92 |
| 40 | -0.01 | -55.93 |
| 50 | -0.02 | -52.06 |
| 60 | -0.03 | -48.91 |
| 70 | -0.04 | -46.24 |
| 80 | -0.055 | -43.93 |
| 90 | -0.07 | -41.9 |
| 100 | -0.086 | -40.09 |
| 150 | -0.19 | -33.15 |
| 200 | -0.34 | -28.9 |
| 300 | -0.75 | -21.66 |
| 400 | -1.29 | -17.21 |
| 500 | -1.94 | -13.98 |

| | | |
|---|---|---|
| 500 | -1.94 | -13.98 |
| 630 | -2.90 | -10.93 |
| 794 | -4.24 | -8.26 |
| 1000 | -6.02 | -6.02 |
| 1260 | -8.26 | -4.24 |
| 1587 | -10.93 | -2.90 |
| 2000 | -13.98 | -1.94 |
| 2520 | -17.33 | -1.27 |
| 3175 | -20.89 | -0.82 |
| 4000 | -24.61 | -0.53 |
| 5040 | -28.43 | -0.335 |
| 6350 | -32.32 | -0.21 |
| 8000 | -36.26 | -0.135 |
| 10.08 K | -40.22 | -0.085 |
| 12.7 K | -44.2 | -0.054 |
| 16 K | -48.2 | -0.034 |
| 20.16K | -52,2 | -0,021 |

| | | |
|---|---|---|
| 600 | -2.67 | -11.54 |
| 700 | -3.46 | -9.66 |
| 800 | -4.3 | -8.17 |
| 900 | -5.15 | -6.98 |
| 1 K | -6.02 | -6.02 |
| 1.5K | -10,24 | -3,19 |
| 2 K | -13.98 | -1.94 |
| 3 K | -20 | -0.915 |
| 4 K | -24.61 | -0.53 |
| 5 K | -28.3 | -0.34 |
| 6 K | -31.36 | -0.24 |
| 7 K | -33.98 | -0.175 |
| 8 K | -36.26 | -0.135 |
| 9 K | -38.28 | -0.107 |
| 10 K | -40.09 | -0.09 |
| 15 K | -47.08 | -0.04 |
| 20 K | -52.06 | -0.02 |

In this filter the cut-off frequency is also the centre-frequency, and what follows, is symmetrical around this. The normal definition of cut-off frequency is useless. The graphs meet where they meet - ruled by the mathematics.
The paper with the dotted line is the target for you to reach, when this version of the filter is used. Keep it reachable, it is all you need. I must emphasise here, that ***in addition of dB-values full level is only reasonably untouched if added levels are below –50 dB***.
It can be a help to know the filter components that are to calculate, but don't expect them to work properly due to unit characteristics. Neither amplitude nor impedance is linear, but the closer to linearity the closer you will come to the calculated values. Remember the need for cancelling the filter function.

**2 way second order.**




For this network Ln = 2 and Cn = 0.5 (n means normalised)

Remark: they are inverse of each other in value
For you to calculate you must know the value of Rdc and the centre frequency (-6.02 dB point in this filter).
Let's say R=8 Ohm and cf=1000 Hz then
L= 2*8/(2*π*1000) H and C=0.5/(8*2*π*1000) F
It is common to have the results in mH and uF, so the value of H must be multiplied with 1000 and the value of F must be multiplied with 1000000.
The calculations can have some practical use, in letting your measuring device draw the curves for you, as a measure of sound level from a perfect loudspeaker, is the same, as a measure of voltage over a resistor with the same value as the loudspeaker. Just remember to use very good parts and avoid electrolytic capacitors.
This version is, as said earlier, perfect. It can be difficult for the unit used, to fulfil the demands, if the difference in size between the units used is too big. A 6-inch bass/mid with 1-inch treble is commonly used, but in my view the difference in size must be decreased.

***It is believed, that one can divide another place in the frequency band and use the same type of filter once more to create a 3-way system. So it is done in the textbooks, but it is very wrong. Every single unit must "know" the existence of the others.***
Filter functions are theoretically developed purely to lowpass filters, and thereby has a well-defined cut-off frequency. With this frequency in mind, it is possible to transform the lowpass filter to bandpass and-or highpass. This method is absolutely correct, if you want a single of these, but ***in a loudspeaker you want them all simultaneously***, so we must have a theory for them all - no matter how many. In practice 3 or 4 way should be enough. I have also developed a 5-way filter, but I don't think I ever will use that.

**The equation that rules them all - 3 ways**
Let us continue to develop the new type of filter, using the same technique as described above, where it led to well known types.
We will now look at expression 1a and 2a of the three previously mentioned

1a. $(s^2 + 2s + 1)/(s^2 + 2s + 1)$
2a. $(s^2 - 2s + 1)/(s^2 + 2s + 1)$

The factor 2 in the part 2s is the inverse of the damping factor of the filter.
The damping factor of this circuit must then be ½ which means, that it as circuit is critically damped. But you are free to decide the size of this damping, so we will change the number to a variable and name it with the letter "a" to look:

**1a. $(s^2 + as + 1)/(s^2 + as + 1)$**
**2a. $(s^2 - as + 1)/(s^2 + as + 1)$**

With these two expressions you again can reach higher order by squaring. This process will lead you to a 5-way system.
I spent 6 years spare time working with that and didn't reach my goal. No matter how

accurate the theory was fulfilled, I still could hear the individual speaker unit.
You can also multiply them with one another, and then you achieve a very interesting result namely

$$\mathbf{H(s) = \frac{s^4 - (a^2 - 2)s^2 + 1}{s^4 + 2as^3 + (a^2 + 2)\,s^2 + 2as + 1}} \quad .$$

This expression is like a Chinese box, as it holds a number of well-known types and a lot of its own.
The minus at the coefficient to $s^2$ in the nominator tells this unit to be connected antiphase to the others, if the value of the expression between the brackets is positive.

**Advantages.**

1. Its summarised amplitude response is always exactly 1
2. The units are in phase with one another at all frequencies if (a) is bigger than or equal to the Square root of 2
3. The step response is for (a) bigger than 2 without ringing
4. Its turn of phase can be chosen as a point to start
5. Its amplitude slope is rounded - slowly reaching its highest slope. The higher (a) value the slower rounding.
6. The units have great overlap (correctional network is unavoidable)
7. When playing it is impossible subjectively to identify the single units as separate sources of sound.
8. It can be realised in full and doesn't demand more of the units than it should be possible for them to deliver - dependant of the (a) value.

**Disadvantages**

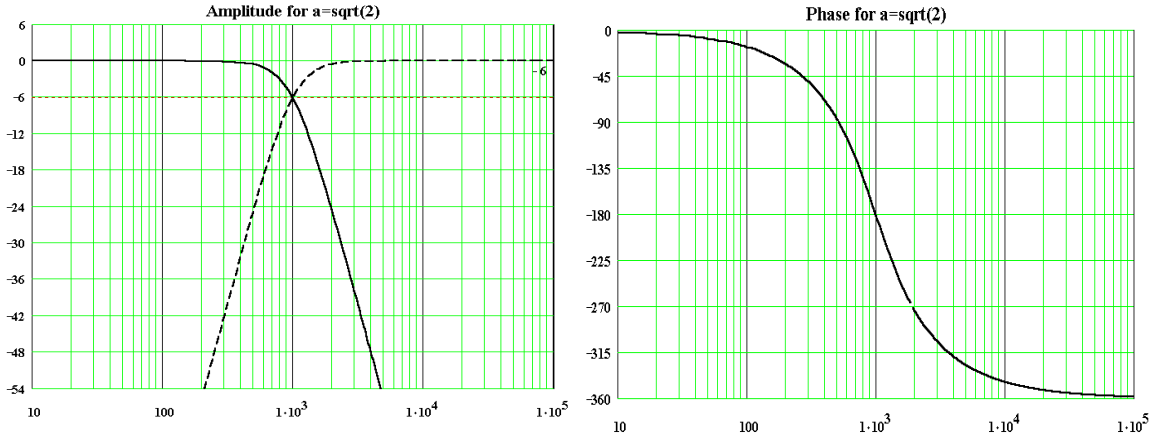
1. The displacement to one another is critical as they work in phase. They must fit within 1 mm from listening position. You are therefore forced to house the units in three separate cabinets, so you can optimise the distance to your listening position. In my views it really is an advantage. It is for you to judge. How you find these positions is described under – "How to build to the limit".

Again all you need is the curvature of the amplitude response for the three units.
But here we have a problem, I'm not to choose - you are. Therefore you must calculate yourself, if you want to see the possibilities.
In general it can be said, that the bass and treble roll off are 24 dB per octave asymptotically ruled by it Q, and likewise the middle but with 12 dB per octave.

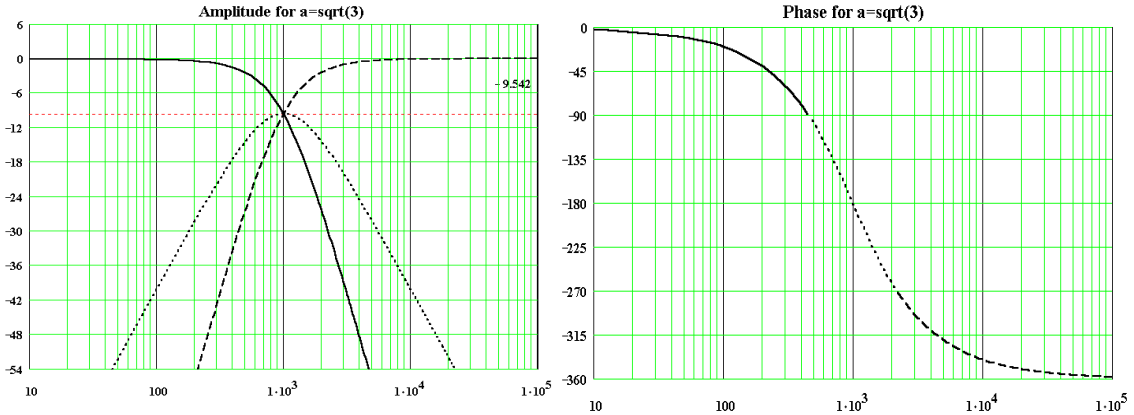
**The factor (a) decides it all.**
The bandpass part $-(a^2-2)s^2$ has to be negative, so (a) has to be greater than or equal to the square root of two.
Let us look at some specific a-values, so you have something to choose from.

**a=1.414214 (square root of 2)**

With this value the bandpass fades away and you get a 2-way 4th order filter.
It is well known as the Linqwitz Riley solution (squared Butterworth).
The Q-value is 0.707 and will give ringing in the step response. This is heard as a focusing on the instrument(s), minimising the recorded sound from the surroundings.



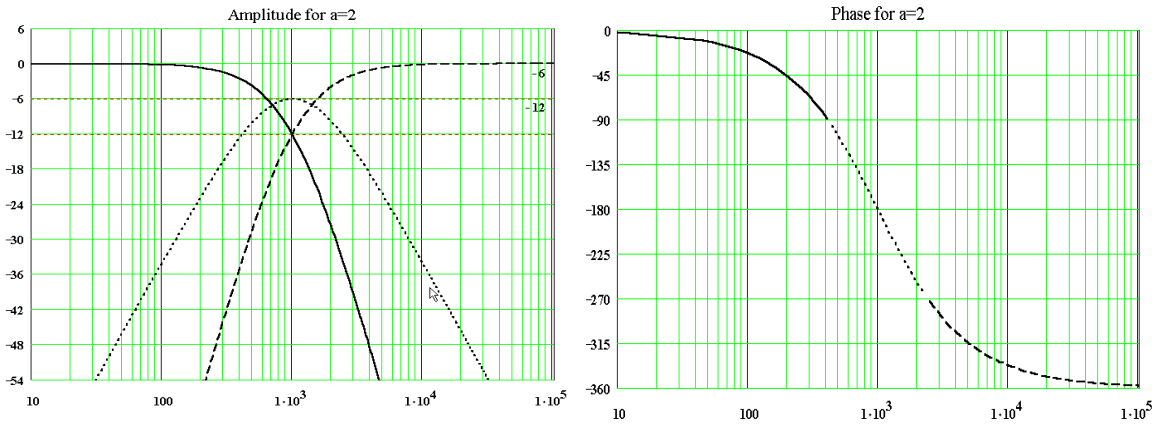

**a=1.732051 (square root of 3)**
This choice gives you a three way Bessel transfer, to my knowledge never seen before, it shows some peculiar characteristics, as the three graphs meet in -9 dB on the centre frequency.
If you are to Bessel, this should be your choice.
NB! The cut of frequency in normal Bessel filters lies 1.732 times higher than the highest frequency with no phase turn. Here it is the centre frequency; so phase turn




starts at Fc/1.732.

**a=2**

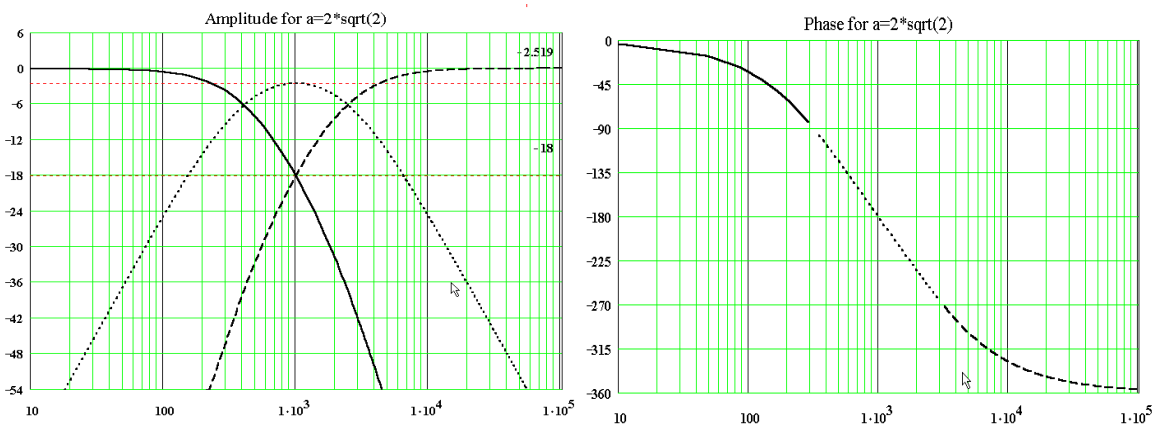
With this value you obtain the sharpest possible cut-off slope obtainable, when you want absolutely no ringing in the step response. The common point for bass and treble is -12 dB and the middle is dampened 6 dB.




Until now we haven't heard so much of the mid-loudspeaker, as it has been dampened and more performed its role as a filler, to add what's missing from summation of bass and treble. The tendency for higher and higher a-values is to tear open the middle part, making room for the middle to come forward, until it fills it all and nullify the whole filter function and it becomes full-range.

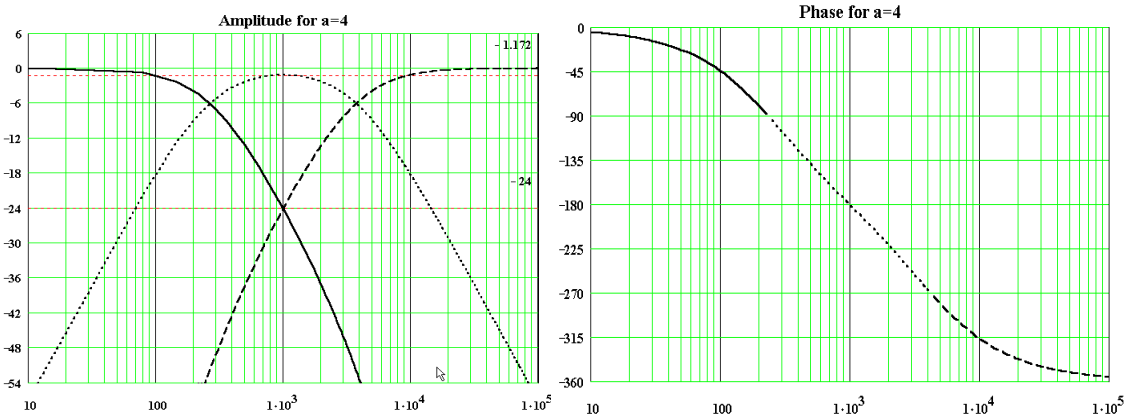
**a=2.828427 (2*square root of 2)**

With that value of "a" you get a rather straight lined phase turn What that means exactly has been subject to discussions, where we couldn't agree. By experiments it




seems as this steady curvature of phase also means a steady sound picture.
Common point for bass and treble at -18 dB and mid is dampened -2.5 dB.

**a=4**




Here the middle band comes forward and the demands for the unit begins to be difficult to fulfil, but there are units out there, to be used with this high value of a.
The phase is disturbed a little, and this disturbance will increase for higher values.

**Calculation of the three amplitude curves.**
To do this you need a calculator.
First we have to change the s to jω in our expression, and put them in two groups: even and uneven exponents, it will look

$$H(j\omega)= \frac{\omega^4 + (a^2-2)\omega^2 + 1}{(\omega^4 -(a^2+2)\omega^2 +1) + (- 2a\omega^3 + 2a\omega)j}.$$

This process changes some signs, but so it is calculating with complex numbers.
The nominator consist of three parts:

$\omega^4$ for the treble - rename to t
$(a^2-2)*\omega^2$ is for the middle - renamed to m
1 is for the bass - renamed to b

The denominator consist of two parts to form a complex number - renamed to

$D1 = (\omega^4 -(a^2+2)\omega^2 +1)$
$D2j = (- 2a\omega^3 + 2a\omega)j$

The dividing into three expressions is shortened down to

$$\frac{t}{D1 + D2j}. + \frac{m}{D1 + D2j}. + \frac{b}{D1 + D2j}.$$

To get amplitude as well as phase, we have to find these expressions as complex

numbers. t+tj and m+mj and b+bj.

$d=D1^2 + D2^2$

| | | |
|---|---|---|
| t=t*D1/d | m=m*D1/d | b=b*D1/d |
| tj= -t*D2/d | mj= -m*D2/d | bj=-b*D2/d |

| | |
|---|---|
| Amplitude treble: | $20*\log (\text{sqrt} (t^2+tj^2))$ |
| Amplitude middle: | $20*\log (\text{sqrt} (m^2+mj^2))$ |
| Amplitude bass: | $20*\log (\text{sqrt} (b^2+bj^2))$ |

Phase: ATAN(tj/j) or ATAN(mj/m) or ATAN(bj/b)

In the expressions ω=1 is the centre frequency, so you just multiply ω with your chosen centre frequency to get the frequency in work. You mustn't let that value be used within the equations.
It is now for you to choose a value of (a) and calculate for different ω-values. To get a reasonable narrow punctuation, let's say 5 points per octave you can multiply 1/64 with $2^{(1/5)}$. The result from this again multiplied with $2^{(1/5)}$ again and again till you reach w=32 and use the results as input for w in the expressions.
You should then have a sufficient number of points to draw the curves on an empty measuring-paper (remember to multiply the used ω-values with your centre frequency).

**To calculate the theoretical filter parts**
The normalised values are only dependent of the value of a
They are called Ln and Cn (inductor and capacitor). To enlighten the termination of the component a further marking is used:
Lns means inductor in series, Cns means capacitor in series
Lnp means inductor in parallel and Cnp means capacitor in parallel.
The expressions are given in normalised form from amplifier towards loudspeaker unit.

| | | |
|---|---|---|
| Bass | $Lns=(2*a^3)/(a^2+1)$ | |
| | $Cnp=(a^2+1)^2/(2*a^3)$ | |
| | $Lns=2*a/(a^2+1)$ | |
| | $Cnp=1/(2*a)$ | |
| Middle | Lns=2/a | This has three different versions |
| | Cns=a/2 | I have chosen the easy one and |
| | Cnp=1/(2*a) | best to control, when not perfect |
| | Lnp=2*a | parts are in use. The two others can |
| | | Be found underneath. |
| Treble | $Cns=(a^2+1)/(2*a^3)$ | |
| | $Lnp=(2*a^3)/(a^2+1)^2$ | |
| | $Cns=(a^2+1)/(2*a)$ | |
| | Lnp=2*a | |

*Alternative configurations of filter components used on bandpass unit.*

*Middle a.*

$Lns=(2*a^3)/(a^2+1)^2$
$Cnp=(a^2+1)/(2*a^3)$
$Cns=(a^2+1)/(2*a)$
$Lnp=2*a$

*Middle b.*

$Cns=(a^2+1)^2/(2*a^3)$
$Lnp=(2*a^3)/(a^2+1)$
$Lns=2*a/(a^2+1)$
$Cnp=1/(2*a)$

*These three versions of bandpass filter have the same slope and phase, but are slightly different in efficiency. These two versions are easier to work with, if your units aren't symmetric.*

Again the Cn and Ln values are normalised to 1-ohm termination and centre frequency 1 Hz. For you to change that, you must decide loudspeaker impedance Z and centre frequency fc.

Then your L= Ln*Z/(2*π*fc) H
C= Cn/(Z*2*π*fc) F

This time the calculated coils and capacitors is of more use, they can of course be altered a little to correct larger tendencies as raising output, but it is advisable to fix that in other ways. Do you want perfect results you have to stick to your calculations.
In order to give you possibility to control your calculations, data is given to the solution preferred by me.

**a=2*sqrt(2)=2.828427**
Dividing frequency is set to 1 kHz.
**1/3 octave**

| Frq | Bass –dB | mid -dB | high-dB | Phase |
|---|---|---|---|---|
| 15.6 | 0 | 56.7 | 144,5 | 5,06 |
| 19.7 | 0 | 52.69 | 136.5 | 6.38 |
| 24.8 | 0.03 | 48.69 | 128.5 | 8.03 |
| 31.3 | 0.05 | 44.69 | 120.5 | 10.1 |
| 39.4 | 0.08 | 40.71 | 112.5 | 12.7 |
| 49.6 | 0.13 | 36.74 | 104.5 | 16 |
| 62.5 | 0.2 | 32.8 | 96,53 | 20,1 |
| 78.7 | 0.32 | 28.91 | 88.62 | 25.3 |
| 99.2 | 0.5 | 25.07 | 80.77 | 31.6 |
| 125 | 0.78 | 21.34 | 73.03 | 39.5 |

| | | | | |
|---|---|---|---|---|
| 157 | 1.21 | 17.76 | 65.43 | 49.1 |
| 198 | 1.85 | 14.39 | 58.05 | 60.6 |
| 250 | 2.79 | 11.31 | 50.96 | 74.1 |
| 315 | 4.11 | 8.62 | 44,25 | 89,4 |
| 397 | 5.89 | 6.38 | 38 | 106 |
| 500 | 8.17 | 4.65 | 32,26 | 124 |
| 630 | 10.98 | 3.44 | 27.03 | 143 |
| 794 | 14.28 | 2.73 | 22.31 | 161 |
| 1000 | 18.06 | 2.5 | 18.06 | 180 |
| 1260 | 22.31 | 2.73 | 14.28 | 199 |
| 1587 | 27.03 | 3.44 | 10.98 | 217 |
| 2000 | 32.26 | 4.65 | 8.17 | 236 |
| 2520 | 38 | 6.38 | 5.89 | 254 |
| 3175 | 44.25 | 8.62 | 4,11 | 271 |
| 4000 | 50.96 | 11.31 | 2.79 | 286 |
| 5040 | 58.05 | 14.39 | 1.85 | 299 |
| 6350 | 65.42 | 17.76 | 1.21 | 311 |
| 8000 | 73.03 | 21.34 | 0.78 | 320 |
| 10.08 K | 80.77 | 25.07 | 0.5 | 328 |
| 12.7 K | 88.62 | 28.91 | 0.32 | 335 |
| 16 K | 96,53 | 32,8 | 0,2 | 340 |
| 20.16K | 104.5 | 36.74 | 0.127 | 344 |
| 25.4K | 112.5 | 40.71 | 0.08 | 347 |
| 32k | 120,5 | 44,69 | 0,05 | 350 |

Filter parts calculated from amplifier towards loudspeaker unit.
Impedance = 5 Ohm, centre frequency 1000 Hz

| bass | | middle | | treble | |
|---|---|---|---|---|---|
| Ls= | 4 mH | Ls= | 0.56 mH | Cs= | 6.3 uF |
| Cp= | 57 uF | Cs= | 45 uF | Lp= | 0.44 mH |
| Ls= | 0.5 mH | Lp= | 4.5 mH | Cs= | 50.6 uF |
| Cs= | 5.6 uF | Cp= | 5.6 | Lp= | 4.5 mH |

**Calculation of step response**
Some modern measuring devices use the step response to calculate from, so here is the expression for the whole system with (a) as only parameter.
At first we have to decide
$\alpha=(-a+\sqrt{a^2-4})/2$
$\beta=(-a-\sqrt{a^2-4})/2$
$S(t)=(-2*a*(\alpha*e^{(\alpha*t)} - \beta*e^{(\beta*t)})/(\alpha-\beta)$

S(t) is the step response dependent on the variation of t.

The drawn curve will look different than normal, as it also shows the turn of phase.
(Remember the units are connected with different polarity)

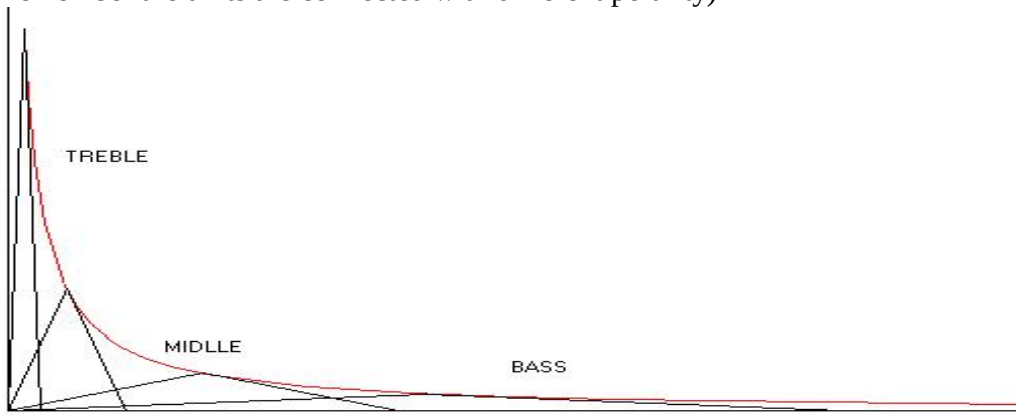



FIGURE. All units are equally phased and will form the familiar response. The area of every single triangle equals the energy present at a given frequency.

**Further development of new filter topology.**
For this we have to go back to the equations from start.

We had 1. $H(s)= (1+s)/(1+s)$ and 2. $H(s)=(1 -s)/(1+s)$
We had 1a. $H(s)=(s^2+as +1)/(s^2+as +1)$
2a. $H(s)=(s^2 -as +1)/(s^2+as +1)$
We again multiply normal version with its allpass version and get:

$$H(s) = \frac{(s^2+as+1)(1+s)(s^2 -as+1)(1 -s)}{(s^2+as+1)(1+s)(s^2+as+1)(1+s)}$$ giving the huge result

$$\frac{- s^6 +(a^2 -1)s^4 -(a^2 -1)s^2 +1}{s^6+(2a+2)s^5+(a^2+4a+3)s^4+(2a^2+4a+4)s^3+(a^2+4a+3)s^2+(2a+2)+1}.$$

This result can be simplified if (a) is substituted with (b-1), it's just a number, to give

$$\mathbf{\frac{-s^6 +(b^2-2b)s^4 -(b^2 -2b)s^2 + 1}{s^6+(2b)s^5+(b^2+2b)s^4+(2b^2+2)s^3+(b^2+2b)s^2+(2b)s+1}.}$$

Which look familiar.
This expression shows again, that the polarity of the four unit have to shift - + - + , the signs of the four parts in the nominator.
We substitute s with jω and rearrange the denominator to give

$$\mathbf{\frac{+\omega^6 +(b^2-2b)\omega^4 4 +(b^2 -2b)\omega^2 + 1}{(-\omega^6+(b^2+2b)\omega^4-(b^2+2b)\omega^2+1)+(2b\omega^5-(2b^2+2)\omega^3+2b\omega)j}.}$$

Which is a 4-way system of 6. Order.

This version has a most interesting slope of phase, when (a) is in the area of 6-7, in the altered expression where b is used 7-8.
"a" decides it all also here.

A=7

I have used this filter once, in a huge construction with the most fantastic result. And it will by used again.
I wasn't happy then, with the use of so many components, even if they were the best of my knowledge at that time. There has to be 6 of the kind on each unit, besides those used to correct the unit's impedance and peaks. It was really overwhelming.
Normally one would say, "forget it", but here we come to the next point of the never-ending story - the components.
If we can develop these with a minimum of errors, or with defects, that at least serves the perception of sound, we are on the right track. For details on that matter you must look under "Components".

***Back to the filter**:*
Calculation of the 4 amplitude curves.
The nominator consist of four parts:
$\omega^6$ for the treble - rename to t
$(b^2-2b)*\omega^2$ is for the high/middle - renamed to hm
$(b^2-2b)*\omega^4$ is for the low/middle - renamed to lm
1 is for the bass - renamed to b
The denominator consists of two parts to form a complex number, renamed to:

$$D1 = (-\omega^6+(b^2+2b)\omega^4-(b^2+2b)\omega^2+1)$$

and $$D2j = (2b\omega^5-(2b^2+2)\omega^3+2b\omega)j$$

The dividing into four expressions is shortened down to

$$\frac{t}{D1 + D2j} + \frac{hm}{D1 + D2j} + \frac{hm}{D1 + D2j} + \frac{b}{D1 + D2j}$$

To get amplitude response as well as turn of phase, we again have to find these expressions as complex numbers t+tj, hm+hmj, lm+lmj and b+bj.

$d=D1^2 + D2^2$

| | | | |
|---|---|---|---|
| t=t*D1/d | hm=hm*D1/d | lm=lm*D1/d | b=b*D1/d |
| tj= -t*D2/d | hmj= -hm*D2/d | lmj= -lm*D2/d | bj=-b*D2/d |

Amplitude treble: $20*\log(\sqrt{(t^2+tj^2)})$
Amplitude |high/middle: $20*\log(\sqrt{(hm^2+hmj^2)})$
Amplitude low/middle: $20*\log(\sqrt{(lm^2+lmj^2)})$
Amplitude bass: $20*\log(\sqrt{(b^2+bj^2)})$

Phase:ATAN(tj/j) or ATAN(hmj/hm) or ATAN(lmj/lm) or ATAN(bj/b)

In the expressions ω=1 is the centre frequency so you just multiply ω with your centre frequency to get the frequency in work. You mustn't let that value be used within the equations.
It is now for you to choose a value of (b=a+1) and calculate for different ω-values. To get a reasonable narrow punctuation, let's say 3 points per octave you can multiply 1/64 with $2^{(1/3)}$. The result from this again multiplied with $2^{(1/3)}$ again and again till you reach ω=32 and use the results as input for ω in the expressions.
You should then have a sufficient number of points- a point at each 1/3 of an octave - to draw the curves on an empty measuring-paper (remember to multiply the used ω-values with your centre frequency).

**To calculate the theoretical filter parts**
The normalised values are only dependent of the value of b and are called Ln and Cn. To enlighten the termination of the component a further marking with numbers is used. See circuit.
The normalised values in letterform, former used, show to be enormous. This time we will go another way, and therefore select a value of (b) to transform the letters to numbers, which is to calculate.
You only have to find the normalised components for the bass, as the normalised components for the other units can be derived from these.
The filter components for the bass can be found from the denominator in s-form by a tricky division, here made easy for you to execute.

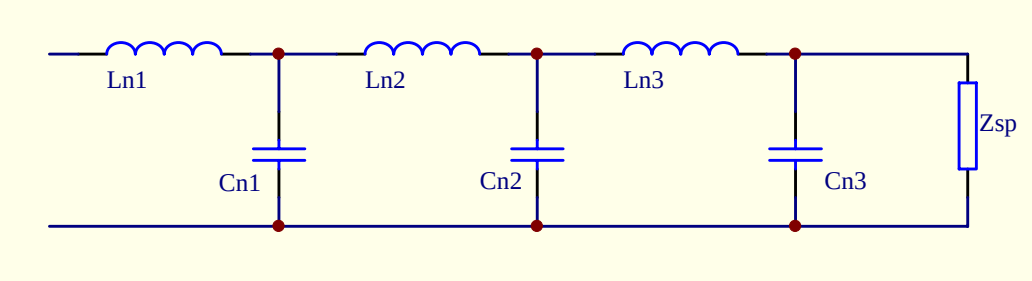
Denominator:
$S^6+(2b)s^5+(b^2+2b)s^4+(2b^2+2)s^3+(b^2+2b)s^2+(2b)s+1$

The coefficients (cp (p for power)) to the different powers of s is transformed to numbers by your choice of value for b.

| | | |
|---|---|---|
| c6=1 ..... | =1 | |
| c5=2*b ..... | = _____ | |
| $c4=b^2+2*b$ ..... | = _____ | |
| $c3=2*b^2+2$ ….... | = _____ | <--- your results |
| $c2=b^2+2*b$ ...…. | = _____ | |

c1=2*b .....…..... = _____
c0=1 .....…... =1

The components drop out from the calculation from unit towards amplifier.




They are named as seen from the diagram. let's calculate:

Cn3=c6/c5 =_____
d4=c4-c3*Cn3 =____
d2=c2-c1*Cn3 =_____
Ln3=c5/d4 =_____
n3=c3-d2*Ln3 =_____
n1=c1-Ln3 =_____
Cn2=d4/n3 =_____
d2=d2-n1*Cn2 =_____ (new d2=former d2)
Ln2=n3/d2 =_____
n1=n1-Ln2 =_____ (new n1=former n1)
Cn1=d2/n1 =_____
Ln1=n1 =_____

Normalised filter components from amplifier towards unit.

| | | |
|---|---|---|
| Bass | Lns=L1 | =_____ |
| | Cnp=C1 | =_____ |
| | Lns=L2 | =_____ |
| | Cnp=C2 | =_____ |
| | Lns=L3 | =_____ |
| | Cnp=C3 | =_____ |
| Low/middle | Cns=C1 | =_____ |
| | Lnp=L1 | =_____ |
| | Lns=L2 | =_____ |
| | Cnp=C2 | =_____ |
| | Lns=L3 | =_____ |
| | Cnp=C3 | =_____ |

| | | |
|---|---|---|
| High/middle | Lns=1/C1 | =_____ |
| | Cnp=1/L1 | =_____ |
| | Cns=1/L2 | =_____ |
| | Lnp=1/C2 | =_____ |
| | Cns=1/L3 | =_____ |
| | Lnp=1/C3 | =_____ |
| Treble | Cns=1/L1 | =_____ |
| | Lnp=1/C1 | =_____ |
| | Cns=1/L2 | =_____ |
| | Lnp=1/C2 | =_____ |
| | Cns=1/L3 | =_____ |
| | lnp=1/C3 | =_____ |

Again these Cn and Ln values are normalised to 1-Ohm termination and centre frequency 1 Hz. For you to change that, you must decide loudspeaker impedance Z and centre frequency fc.

Then your L= Ln*Z/(2*π*fc) H

C= Cn/(Z*2*π*fc) F

This time the calculated coils and capacitors are of more use due to their turn of phase, as their effect on level are difficult to measure precisely.

You must be very skilled with mathematics to subtract the inside components of the loudspeaker from the filter. But by measuring with your microphone placed very close to the unit, it should be possible to take away the last 2 filter components from high/middle and treble. This because the units themselves have 12 dB roll of.

If you have peaks in the frequency band you must correct them in a way so you keep the impedance linear - see “How to build to the limits” -, this takes 6 components, so units used should be selected with care.

In order to give you possibility to control your calculations, data is given to a solution.

b=7

Dividing frequency is set to 1 kHz.

**1/3 octave**

| frq | bass -dB | lowmid -dB | highmid-dB | treble-dB | Phase |
|---|---|---|---|---|---|
| 15.6 | 0.07 | 42.5 | 114 | 217 | 11.9 |
| 19.7 | 0.1 | 38.5 | 107 | 205 | 15 |
| 24.8 | 0.16 | 34.6 | 98.8 | 193 | 18.8 |
| 31.3 | 0.26 | 30.6 | 90.8 | 181 | 23.6 |
| 39.4 | 0.41 | 26.8 | 83 | 169 | 29.7 |
| 49.6 | 0.64 | 23 | 75.2 | 157 | 37.1 |
| 62.5 | 1.0 | 19.3 | 67.5 | 145 | 46.2 |
| 78.7 | 1.53 | 15.9 | 60 | 134 | 57.3 |
| 99.2 | 2.33 | 12.6 | 52.8 | 123 | 70.4 |
| 125 | 3.48 | 9.77 | 45.9 | 112 | 85.6 |
| 157 | 5.05 | 7.33 | 39.4 | 101 | 103 |
| 198 | 7.12 | 5.38 | 33.5 | 91.4 | 121 |
| 250 | 9.71 | 3.97 | 28 | 82 | 141 |
| 315 | 12.8 | 3.07 | 23.1 | 73 | 161 |
| 397 | 16.5 | 2.69 | 18.7 | 64.6 | 182 |
| 500 | 20.6 | 2.8 | 14.8 | 56.7 | 203 |
| 630 | 25.2 | 3.42 | 11.5 | 49.3 | 225 |
| 794 | 30.4 | 4.58 | 8.59 | 42.4 | 247 |
| 1000 | 36.1 | 6.3 | 6.3 | 36.1 | 270 |
| 1260 | 42.4 | 8.59 | 4.6 | 30.4 | 293 |
| 1587 | 49.3 | 11.5 | 3.4 | 25.2 | 315 |
| 2000 | 56.7 | 14.8 | 2.8 | 20.6 | 337 |
| 2520 | 64.6 | 18.7 | 2.69 | 16.5 | 358 |
| 3175 | 73 | 23.1 | 3.07 | 12.8 | 379 |
| 4000 | 82 | 28 | 3.97 | 9.71 | 399 |
| 5040 | 91.4 | 33.5 | 5.38 | 7.12 | 419 |
| 6350 | 101 | 39.4 | 7.33 | 5.04 | 437 |
| 8000 | 112 | 45.9 | 9.77 | 3.48 | 454 |
| 10.08 K | 128 | 52.8 | 12.6 | 2.33 | 470 |
| 12.7 K | 134 | 60 | 15.9 | 1.54 | 483 |
| 16 K | 145 | 67.5 | 19.3 | 1 | 494 |
| 20.16K | 157 | 75.2 | 23 | 0.64 | 503 |
| 25.4K | 169 | 83 | 26.8 | 0.41 | 510 |
| 32k | 181 | 91 | 30.6 | 0.26 | 516 |

Filter parts calculated from amplifier towards loudspeaker unit.
Impedance = 6 ohm (all units) centre frequency 1000 Hz and b=7

| bass | low/middle | high/middle | treble |
|---|---|---|---|
| Ls=11.43 mH | Cs=120uF | Ls=0.211mH | Cs=2.216 uF |
| Cp=120uF | Lp=11.43mH | Lp=2.216uF | Lp=0.221mH |
| Ls=1.697 mH | Ls=1.697mH | Cs=14.92uF | Cs=14.92 uF |
| Cp=15.16uF | Cp=15.16uF | Lp=1.67mH | Lp=1.67 mH |
| Ls=0.243 mH | Ls=0.243mH | Ls=104.2uF | Cs=104.2 uF |
| Cp=1.895 uF | Cp=1.89uF | Cp=13.36mh | Lp=13.36mH |

**Calculation of step response**
Some modern measuring devices use the step response to calculate from, so here is the expression on step-response for the whole system with (b) as only parameter.
At first we have to decide
$\alpha=(-b+sqrt(b^2-4))/2$
$\beta=(-b -sqrt(b^2-4))/2$
$L=((\alpha-1)*(\beta-1)*(-2))/((-\alpha-1)*(-\beta-1))$
$N=(2*\alpha*(\alpha+\beta)*(\alpha-1))/((\alpha+1)*(\alpha-\beta))$
$P=(2*\beta*(\alpha+\beta)*(\beta-1)*((\beta-\alpha)*(\beta+1))$
Then
$S(t)=L*e^{(-t)}+N*e^{(\alpha*t)}+P*e^{(\beta*t)}$

S(t) is the step response dependent on the variation of t from zero to?
Again the step response is without trigonometric functions and thereby without ringing.

**Even further**
For those out there, who as I find it amusing to search into the unknown, there may be surprises to find in the following expression.

$$H(s)= \frac{(s^2+as+1)(s^2-as+1)(s^2+bs+1)/(s^2-bs+1)}{(s^2+as+1)(s^2+as+1)(s^2+bs+1)(s^2+bs+1)}$$

Here you have the possibility of working with two different Q´s and use (a) and (b) values that turn of some of the units.
There may be other ways, other basic equation to work with, than I have chosen.
Could this paper serve as inspiration for others to look into that area, I would have gained much from my work.
I'm not a great master of mathematics, as I much more am a person of practical skill equipped with an annoyingly sharp hearing too.
All I want is, that what is gained in practice must be expressed mathematically, otherwise you have gained nothing.
That is why, I have enormous concern about expressing myself, when it comes to wires, components and the human hearing. But I am not alone in this respect, even acoustic engineers face the same problem. There is no exact science in this department for now.
In the following you must forgive my use of experiences from listening.
Thereby I will enter into a strange world governed by my hearing, intuition, taste, feelings and even quantum mechanics that I really don't like.
But there has been no way around that, as all my experiences with sound have happened in my brain. That part of it all is for me to see the most important one. May yours work like mine.